THE WILL TO MATHEMATICS: MINDS, MORALS, AND NUMBERS

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INTRODUCTION.

The 1990s was The Decade of Sociology in mathematics education. The sociology of mathematics has become a core ingredient of discourse in mathematics education and the philosophy of mathematics and mathematics education. Unresolved questions and uncertainties have emerged out of this discourse that hinge on the key concept of social construction. More generally, what is at issue is the very idea of "the social." Within the framework of the general problem of "the social," we want to open a discussion of boundaries and margins in mathematics and mathematics education. By theorizing the divisions of purity and danger, we will be able to better understand the intersection of logic, mathematics, and thinking with gender, race, and class, and morals, ethics, and values in the classroom.

The process of transforming the sociology of mathematics and the sociology of mind into pedagogical tools for mathematics educators and

philosophers of education has already begun. One of the tasks before us is the development of a more profound and at the same time more practical grasp of "the social." Our objective in this paper is to move ourselves as well as our readers in the direction of just such a grasp of the social.

The Will to Mathematics

By "The Will to Mathematics," we mean the drive to see and feel in mathematics something pure, transcendent, and certain with results that approach a level of truth as high as humans can hope to achieve. Our claim is that the reality of mathematics lies in discourse, so mathematics is as real – and only as real – as ordinary social life. To pursue this claim, let us begin by exploring what mathematics represents.

To represent something is to construct something in symbols or images to stand for it. We are going to pursue this idea for mathematics without addressing the technical subtleties of accurate representation as a problem in philosophy. When we ask, "What does mathematics represent?" we are asking, "What do math symbols and images stand for, what do they represent?" What *might* they stand for? For simplicity, we are going to continue by using the idiom, "What *are* mathematical objects?"

There is an interesting mythology about mathematics and mathematicians that reinforces ideas of purity and genius, and even madness: we are sure Edmund Landau is not the only mathematician who has claimed that "Wir Mathematiker sind alle ein bi*B*chen meschugge." We have in mind stories about, for example,

non-Euclidean geometry. Dirk Struik (1967: 167), mathematician, historian of mathematics, and perhaps the first person to use the phrase "sociology of mathematics" in a publication, wrote about non-Euclidean geometry:

> It is remarkable how the new ideas sprang up independently in Gottingen, Budapest, and Kazan, and in the same period after an incubation period of two thousand years. It is also remarkable how they matured partly outside the ge ographical periphery of the world of mathematical research.

And the equally distinguished historian of mathematics, Carl Boyer (1968: 585), found in this case a "startling…simultaneity of discovery" by three men, "one German, one Hungarian, and one Russian." Yet, as even the most cursory review of the history and social networks of these discoveries clearly demonstrates, there is nothing either startling or remarkable about this case (Restivo, 1983: 232-235). Could scholars of the caliber of Struik and Boyer have been unaware of these connections, or were they bound by some code to present the case in a rhetoric of mystery and transcendence? And consider the case of the famous number, 1729. This is, as many of you may recognize, the number of the taxi that G. Hardy rode in on his way to visit a hospitalized Ramanujan. He tells Ramanujan that the taxi had "a rather dull number." Ramanujan replies that on the contrary it is a very interesting number. It is the smallest number that is the sum of two cubes in two different ways. Now this is as much as we had ever read about this episode, and it is easy to come away with the notion that this genius had on the spot and virtually instantly recognized this feature of 1729. But Ramanujan's biographer, Robert Kanigel (1991: 312), makes the more plausible

claim that Ramanujan had noticed or come across this quality of 1729 years earlier, recorded it, and remembered it. It is not incidental that neither Hardy nor Ramanujan mentioned any of the other ways the number 1729 is interesting.** But the story that reinforces the sense of genius is a much better story for mathematics, isn't it? It is a story repeated for many instances of mathematical genius and even occurs in the cases of idiot savants. In these cases, the mystery of genius is substituted for the mundane quality of hard, even obsessive work, recording, memorization, and remembering (Dehaene, 1997: 167-172). Why are these good stories for mathematics?

Mathematics might represent a reality, or a mathematical reality that lies "outside us." Hardy (1967: 123-24, 130), for example, wrote:

> I believe that mathematical reality lies outside us, that our function is to discover or observe it, and that the theorems which we prove, and which we describe grandiloquently as our creations, are simply our notes of our observtion.

Hardy was a Platonist. Michael D. Resnik (1993: 39), a contemporary Platonist, describes Platonism as the idea that

> Numbers, sets, functions, and other paradigmatic mathematical objects are …outside spacetime and incapable of interacting with ordinary bodies within it.

Platonism refers to Plato's notion that the objects of our sensory experience are reflections of ideal non-spatio-temporal "forms." Sometimes, Platonism is used to label the idea that mathematical objects are "real" (Schechter, 1998: 113). As social theorists, we wish to know the function of labelling mathematics "transcendent." What is the consequence of defining mathematics outside of "ourselves," why do we do this?

We took the bold step recently of re-reading Plato, along with Huntington Cairns' "Introduction" to *The Collected Dialogues* (Hamilton and Cairns, 1989). "Parmenides" is interesting because it has caused scholars great difficulty: first, because the dialogue is one of most resistant to reasoned interpretation and second, because the close scrutiny of the idea of the "forms" can leave one wondering just what Plato had, so to speak, in mind and how satisfied he was with the very idea (Hamilton and Cairns, 1989: 920). The whole idea is left in doubt at the end of Parmenides' critique. Cairns (in Hamilton and Cairns, 1989: xviii-xix), who has explored this issue with an expertise that we lack, concludes that Plato did indeed believe that the "forms" or "Ideas" exist outside of our minds. But he also suggests the concept may be more earthly than ethereal, in some ways kin to "naturalism, pragmatism, positivism, analysis, and existentialism." Could Platonism be infected with a relatively benign form of the social construction virus?

Even if one can find hints of social theory in Platonist views of mathematics, the image of something "outside" of us – something transcendent, godlike, pure, abstract – keeps mathematics ultimately separated from the social and material realms of experience. For the social theorist, references to realms "outside" of us are understood as pointing to social referents. Emile Durkheim (1995), and George H. Mead (1947) pioneered in the development of social theory as the rejection of transcendence, immanence, and psychologism.

The peripatetic mathematician known for his "open brain," the late Paul Erdos, wrote that "There's an old debate about whether you create mathematics or just discover it. In other words, are the truths already there, even if we don't yet know them? If you believe in God, the answer is obvious. Mathematical truths are there in the SF's [Supreme Fascists's] mind, and you just rediscover them…." (in Hoffman, 1998: 26).

There was a young man who said, 'God,
It has always struck me as odd
That the sycamore tree
Simply ceases to be
When there's no one about in the quad.'

'Dear sir, Your astonishment's odd;
I am always about in the quad:
And that's why the tree
Will continue to be,
Since observed by,
Yours faithfully, God.

Mathematics might also represent God or a religion. In Mesopotamia, the ratio 2/3 was deified as the god Ea the Creator. The mathematical properties of certain numbers make them candidates for representing deities. "7" was a symbol for the sacred world in Mesopotamia. The Hebrews rejected the practice of deifying numerals. Isaiah 44:6 is sometimes cited as an exception; but this is a bit tricky. In this passage, the Lord says "I am the first and I am the last" (in the closing paragraphs of the New Testament, these words appear again when the Lord says: "I am the Alpha and the Omega, the first and the last, the beginning and the end" (Rev. 22:13). Generally, insofar as mathematics has historically been the science of the infinite it has been the science of God. There are many

other examples we could point out, but the important point is that in the case of God as in the case of the "forms," mathematics represents a transcendental realm.

In spite of the widespread support in mathematics and the philosophy of mathematics for Platonism, supporters have not been able to escape the self-contradiction, and even the absurdity, of the transcendence claim. "How," muses Paul Benacerraf (in Schechter, 1998: 52), "if mathematical knowledge stands outside of space of time…can [it] be reached from an earthly realm deeply submerged in space and time?" The only answer that doesn't mangle logic and social understanding must be some variation of "Das ist nicht Mathematik, das ist Theologie" (Paul Gordan's famous reply to Hilbert during their invariant theory proof war).

It is interesting to note that Paul Erdos, who claimed that "A mathematician is a machine for turning coffee into theorems," behaved in opposition to this idea as if in fact mathematics was "a social activity, a movable feast" (Schechter, 1998: 14). The idea that mathematics as a vocation is social would not be disputed by many working mathematicians. The trouble begins when the sociologist wants to draw out the more technical meanings of "social". Furthermore, the sociologist wants to press the idea of the "social" beyond its everyday meaning and to argue that mathematical objects themselves are social. The transcendental realm is a cultural creation, not a reality out of space and time. So is the supernatural, and so then are the gods and God. The fundamental project

of the sociological sciences can indeed be viewed as locating the everyday world referents for the transcendental and supernatural.

What can we conclude as social constructionists, that is, as sociological realists? The idea that mathematics is pure or transcendent is "an expression of the felt autonomy of the inner activities of the intellectual network" (Collins, 1998: 878). The certainty of mathematics is a function of how tightly the generational links across mathematical networks are interwoven. The "chain of social conventions" in mathematics is robustly repeatable. It is this robustness that accounts for the sense of certainty mathematicians and laypeople alike share about mathematics.

Neither truth, certainty, nor thought itself "arise in isolated brains or disembodied minds" (Collins, 1998: 877). They all arise in social networks. At the end of the day, sociologists are wont to ask "How could any of these phenomena arise anywhere else; what is there that is anywhere else?" It is discourse, with its "objective, obdurate quality," that produces that "strong constraint that answers the concept of truth" (Collins, 1998: 865). Even the most elementary exercise in mathematics, indeed even the most elementary understanding of an equation, engages us in a form of discourse (and more broadly, in Wittgenstein's terms, a form of life), a network of teachers and students, of researchers, inventors, and discoverers. The "universality" of mathematics, like the universality of any cultural system, trait, or representation is grounded in the universality of its use.

Toward an Archaeology of Mathematics

It may appear that we want to reject outright the certainty, purity, and universality of mathematics. But we can be more modest and seek only to disturb the tranquility with which these notions are accepted (Foucault, 1972: 23-26). We would then want to show that mathematics does not come about of itself, but that it is constructed. If it is constructed, there must be rules of construction, and these must be known. Our project from this perspective is to bring the justifications for doing and teaching mathematics to the surface and to scrutinize them.

What are the conditions under which it is reasonable and legitimate to do, use, apply, and teach mathematics? Are there things about mathematics, including mathematics itself, that we might want to consider discarding because they are illusions, illegitimate constructions, or ill-acquired? Should we never use them, draw on them temporarily, store them for possible future use? Is it enough to simply remove mathematics from its throne of purity?

Whether we are modest or immodest in our methods and theories, as soon as we question (for whatever reasons?) the unity, purity, and universality of mathematics, "it loses its self-evidence; it indicates itself, constructs, only on the basis of a complex field of discourse" (Foucault, 1972: 23-24). Foucault, of course, was not thinking of mathematics here. He treated mathematics as something of a special case, immune to the power of his archaeological method.

Do numbers hide something? Are they embedded in networks of power, and are they deployed in ways that purposefully obscure the power behind their visual and oral re-presentations? (cf., Said, 1983: 184)? How is it then that mathematics seems to have escaped matter (the biologist Scott Gilbert, in Schiebinger, 1999: 162)? How has it hidden the fact that it is a discipline that disciplines? Mathematics is an everywhere dense discourse. How do we reveal the systems of regularities that determine mathematicians by determining their situations, functions, perceptions, and practical possibilities? How do we reveal the social, cultural, and historical conditions that "dominate and even overwhelm" mathematicians (cf. Foucault, 1970: xiii-xiv)?

Mathematics, like any discourse, like any language, "is to some degree a jargon, but it is also a language of control and a set of institutions within the culture over what it constitutes as its special domain" (Said, 1983: 219). We need, following Foucault's method, to be able to reconceptualize the problem of mathematics not as a problem in ontology (or even in classical epistemology) but as a problem in politics and ethics (or as we will put it here, a problem in morals). Let us pursue this Nietzschean turn.

The Morality of Mathematics.

Individuals do not make decisions about what is right and wrong or true and false on their own. Such decisions are settled by institutions (cf. Douglas, 1986: 4). "Classification, logical operations, and guiding metaphors are given to the individual by society" (Douglas, 1986: 10). It is on the basis of such

Durkheimian considerations that sociologists of knowledge of our type reach the conclusion that mathematics is a moral system. It is useful to consider, at this point and in some detail, Durkheim's (1961: 29-30) remarks on the categories of space, time, and causality:

> They represent the most general relations which exist between things; surpassing all our other ideas in extension, they dominate all the details of our intellectual life. If men do not agree upon these essential ideas at any moment, if they did not have the same conceptions of time, space, cause, number, etc., all contact between their minds would be impossible, and with that all life together. Thus, society could not abandon the categories to the free choice of the individual without abandoning itself….There is a minimum of logical conformity beyond which it cannot go. For this reason, it uses all its authority upon its members to forestall such dissidences…. The necessity with which the categories are imposed upon us is not the effect of simple habits whose yoke we can easily throw off with a little effort; nor is it a physical or metaphysical necessity since the categories change in different places and times; it is a special sort of moral necessity….

We should explore number's unique role in molding our conceptions of abstraction, purity, and the sacred, and its primacy in constructing relations, separations, and boundaries between minds and bodies.

The moral necessity of mathematics is enhanced as the professional boundaries are constructed and concretized around those thought communities and thought collectives (Fleck, 1979/1935) dedicated to these very ideas. Foremost among these communities and collectives are the mathematicians.

All institutions provide the categories of thought, set the terms for knowledge and self-knowledge, and fix identities. But more than this, they "must secure the social edifices by sacralizing the principles of justice" (Douglas, 1986:

112). In mathematics, classifications and theories, proofs and conjectures are held together by the sacred glues of logic and reason. Given this conception of the nature and function of institutions, it should not be surprising to find that questions and issues of morals merge with questions and issues of what is real and what is illusory. We are left with the following question: how does classroom practice change if we understand that problems of truth and falsity, what is right and what is wrong, are moral problems? What would it mean to address our classroom practices in this context?

The Word or the Act?

What, "In the beginning was the Word?" Absurd.
Then maybe it should say "In the beginning was the Mind?"
Or better "…there was Force?"
Yet something warns me as I grasp the pen,
That my translation must be changed again.
The spirit helps me. Now it is exact.
I write: "In the beginning was the Act."

Thus did Goethe (1963 :153) have Faust speak. No one was clearer and more elegant in locating the social sources of ideas, words, and mind than Marx (1958: 104):

> Even when I carry out *scientific work,* etc., an activity which I can seldom conduct in direct association with other men – I perform a *social,* because *human,* act. It is not only the material of my activity – like the language itself which the thinker uses – which is given to me as a social product. My *own* existence *is* a social activity.

Then early in this century, Emile Durkheim and Marcel Mauss (1963) demonstrated that ideas and concepts in "primitive societies" arise from and reflect social structures, networks of human beings interacting in conflict and cooperation. Here are the beginnings of what we commonly refer today as social

construction theory. It is important to clarify the nature of this theory before we go on, since it has generated so much confusion even among its advocates. The very idea that science and mathematics are socially constructed has generated the Science Wars, and made sociologists of science targets for physical scientists who have labelled us with Quine's most pejorative epithet, "anti-science." When our anti-science image is coupled to our image as relativists, we become a danger to the very foundations of Western civilization. But anti-science and relativism are not necessary ingredients of social constructionism. Durkheim (1961: 31-32) himself already remarked that

> From the fact that the ideas of time, space, class, cause or personality are constructed out of social elements, it is not necessary to conclude that they are devoid of all objective value.

The most unsettling pseudo-deduction from social construction theory is that it eliminates the possibility of telling the truth. If postmodernism has eliminated the possibility of telling the truth, or at least made telling the truth problematic, it has done so by masking the truths of sociology and anthropology. Social construction theory must be in a sense turned on itself in order to eliminate pseudo-deduction monstrosities. For in fact, as Dorothy Smith has so elegantly pointed out, it is just social construction theory that makes telling the truth possible. Reference and representations are social activities and processes. Following Mead (1938, 1947) and Bakhtin (1981, 1986), Smith (1996: 193-195) argues that "a fully social, dialogic account of knowledge and truth holds out for systematic inquiry the possibility of telling the truth about what it finds." Truth and knowledge, as fallible and tentative achievements, are manufactured by

human beings who accomplish what they know and what they can know in common (cf. Fleck, 1979, on thought collectives).

The Public Understanding of Mathematics.

Ancient and esoteric debates and dialogues about the nature of mathematics have recently spilled over into the public domain of mathematics. The February 10, 1998 issue of *The New York Times* carried the following headline: "Useful Invention Or Absolute Truth: What is Math?" The author, George Johnson (1998: 1), reviewing a recent book by mathematician Reuben Hersh, writes:

> Dr. Hersh's book is one of several recent works contending that mathematics is not an ethereal essence but comes from people who invented, not discovered it. The sentiments presented in the books are not entirely new and the mathematical puzzle has hardly been solved. But the idea of a human-centered mathematics may be gaining force and respect.

The authors Johnson cites in sketching the idea of a "human-centered mathematics" are all "working mathematicians and scientists, ***not postmodern critics viewing the territory from afar"*** (our emphasis):

> They emphatically reject those who try to dismiss mathematics and science as arbitrary constructions, or white male Eurocentric folklore. But they are just as adamant in rejecting what most mathematicians and many scientists have come to take for granted: the Platonic creed.

A boxed insert announces that "Some scholars say mathematics emerged from the inferior parietal cortex, not a Platonic ether."

Now let's review these excerpts and tease out what the author has accomplished in this article. First, it looks like he has advanced the public's understanding of mathematics in terms we would all advocate to one extent or

another: mathematics is invented, not discovered; it is human-centered. But in fact he has reinforced the pernicious idea that physical and natural scientists and mathematicians are the ultimate authorities on how the world works, and even on how the human-centered world works. The human, social, and cultural sciences and humanities are summarily dismissed under the rubric of postmodern scholarship which can only study science and mathematics from "afar." Not one sociologist's or anthropologist's name appears in the article, even though it is easy enough to find such names in the writings of mathematicians like Reuben Hersh (who cites, for example, both David Bloor and Sal Restivo).

If articles like this advance in some way the public's understanding of mathematics, they do so in a way that masks the public's understanding of just those scholarly endeavors that have facilitated human-centered ways of thinking about mathematics. In the end, articles like this promote the authority of a traditional hierarchy of inquiry that legitimates the physical and natural sciences and delegitimates the social sciences and humanities. That authority extends not only to reflections on the social nature of science and mathematics, but to the very heart of the subject matter of the social sciences and humanities – including the study of religion, God and gods, the soul, consciousness, mind, and thought.

Furthermore, it is clear that programs to advance the public understanding of mathematics (along with science more generally) are about the technical content of mathematics. Mathematicians, public officials, and already understanding publics want people to be able to do the mathematics they need to

be able to do in order to achieve the objectives of their governments and immediate employers. They want people to participate in the social and moral order of educated publics. They want women and minorities to share in the mathematical skills needed to keep society running (without much consideration for the differential benefits that accrue to members of that society). And at some level they would like the public to understand mathematics and mathematicians in an appreciative sense. The social sciences and humanities enter this public understanding project primarily in the aborted, truncated and crippled forms internalized by mathematicians and philosophers. The result is that the actual voices of the social scientists and humanities scholars are silenced and the alleged dangers of legitimizing their inquiries are minimized. Public understanding of mathematics with a human face then comes to be about demonstrating the limits of mathematics and science, the ease of learning these traditionally esoteric subjects, and the all too human qualities of their practitioners. The point of the public understanding programs should go beyond understanding and appreciating mathematics and mathematicians to encompass their social grounds, roots, forms, and functions. We don't want to argue for eliminating the voices of the mathematicians and philosophers in advancing the public understanding of mathematics in the broader sense we have been discussing. We do want to open up opportunities for social scientists and humanities scholars to play a more visible role in advancing the public understanding of mathematics.

Social Construction Unbound: What it Really Means.

Mathematical knowledge is not simply a "parade of syntactic variations," a set of "structural transformations," or "concatenations of pure form" (adapted from Geertz, 1983). Mathematical forms or objects represent – are – sensibilities, collective formations, and worldviews. The foundations of mathematics are not located in logic or systems of axioms but rather in forms of life. Mathematics embodies mathematical worlds, and mathematical worlds are configured by societal and cultural worlds. The more professionalized mathematics becomes, the more it embodies itself, its own world of professional objects. This is the source of that mysterious sense of beauty and transcendence that infects mathematicians and philosophers. It is caused by the difficulty of locating everyday referents for mathematics. The situation is analogous to why God and the gods are located outside of ourselves instead of within our social formations.

Explaining the content of mathematics is not a matter of constructing a simple causal link between a mathematical object such as a theorem and a social structure. Jean Dieudonne's (Nordon, 1981) challenge exposes a fundamental and widespread misunderstanding about sociological claims and theories:

> Celui qui m'expliquera pourquoi le milieu social des
> Petites cour allemandes du XVIII siecle ou vivait
> Gauss devait inevitablement le conduire a s'occuper
> de la construction du polygone regulier a 17 cotes,
> eh bien, je lui donnerai une medaille ou chocolat.

Dieudonne's error is to imagine that only "external" milieux hold social influences. This is of a piece with the idea that the term "social," as in "social construction," is a synonym for "political," "religious," "economic," or

"ideological;" and that it means, essentially, "false" or "arbitrary." We want to remind the Steven Weinbergs, Lewis Wolperts, and Richard Dawkins that to say that science and mathematics are socially constructed is ***not*** to say they are false, arbitrary, fabricated out of thin air, or the direct product of external political, religious, economic, or ideological forces, causes, or influences. The sociological task is not to make such claims but rather to unpack the social histories and social worlds embodied in, for example, mathematical objects. The objective of this "unpacking" should be to allow us to move more freely in the world we have collectively created.

Mathematical objects are things produced by, manufactured by, social beings through social means in social settings. There is no reason why an object such as a theorem should be treated any differently in this sense than a sculpture, a teapot, a painting, or a skyscraper. Only alienated and alienating social worlds could give rise to the idea that mathematical objects transcend time and space. Mathematicians work with notations, symbols, and rules; they have a general reservoir of resources, a toolkit, socially constructed around social interests and oriented to social goals. The objects they construct take their meaning from the history of their construction and usage, the ways they are used in the present, the consequences of their usage inside and outside of mathematics, and the network of ideas they are part of within math worlds and within larger societal worlds.

Let us take you back to the beginning of this paper and ask you to fill in the blanks in the following paragraph:

> **To conceive _____ is difficult; and to describe ____ is impossible, even if one is able to conceive _____. For it is not easy for that which is imperfect to apprehend that which is perfect, and it is hard for that which is of short duration to have dealings with that which is everlasting. The one ever is, the other passes; the one is real, the other is but shadowed forth by sense-picturing. So widely is that which is mortal separated from that which is divine. And the wide interval between them dims men's vision of the Beautiful. With our eyes we can see bodies; but that which is incorporeal and invisible and without shape, and is not composed of matter, cannot be apprehended by senses such as ours…***

This is from the Hermetica, and the correct way to fill in the blanks is with the words "God" and "Him." Now consider that the excerpt would have made sense if we had filled in the blanks with the word "mathematics.

Now consider this description of mathematics in Don DeLillo's (1982: 164) novel, The Names:

> "There is no test," Charles said. "The only test is mathematics. You've got to know the secrets. Look at him. He speaks to no one. He says he's not able to talk about it. There are certain things he can't discuss with his *professors*. It's too bloody rarefied. It makes no sense if you don't know the secrets, the codes. It means nothing, is in fact absolutely useless….It doesn't bear on human experience, human progress, ordinary human language….It's interesting in itself, you see. It refers to itself and only itself. It's the pure exercise of the mind. It's Rosicrucianism, druids in hoods. The formal balances, that's what counts. The patterns, the structures. It's the inner consistencies we have to search for. The symmetries, the harmonies, the mysteries, the whisperies.

Is it any wonder that Durkheim links God and logic in his classic treatise, and reveals their nature as collective representations with a little sociological insight?

In the final pages of *The Elementary Forms of Religious Life,*** Emile Durkheim turns his attention to the sociology of logical concepts. God and logic are linked because they seem to escape the bonds of space and time. Durkheim

connects them because in spite of what appear to be transparently transcendental and universal qualities, they are both collective representations and collective elaborations – social constructions. Durkheim was one of the first social theorists to recognize that all ideas and concepts, however abstract or irrational, have what we have called "standard" referents. To the extent that we invoke transcendent, supernatural, and mystical explanations, we invoke non-standard referents. The term "God" is, in terms of reference, paradigmatically non-standard. The term "tree" is, by contrast, paradigmatically standard. This distinction between standard and non-standard is only useful as a temporary heuristic device. All referents are standard. Durkheim's great discovery (a consolidation of centuries of intellectual moves) was to find the standard referent for God, Society. Logical concepts, like religious ones, are collective representations. In general, mental facts are social facts. Mathematical objects, Just like other objects (symbolic and material), are collective representations and collective elaborations.

Every emancipation restores the social world and social relationships to ourselves, to paraphrase Marx. Social constructionism (sociology) is just such an emancipation. It de-alienates and de-fetishizes representation, reference, cognition, knowledge, and belief. This is not simply an intellectual emancipation, but a political one.

Conclusion.

How, now, are we going to answer the questions,

"What does mathematics represent?"
"What are mathematical objects?"

"What is mathematics, really?" (Hersh's [1997] query).

The most lucid, empirically grounded study of ideas as social constructions is Randall Collins' recently published The Sociology of Philosophies.

Randall Collins, The Sociology of Philosophies, Harvard, 1998.

Ideas = Communication = Social Interaction.
Personalities = The network processes which bring them to our attention as historical figures.
Creativity builds up in intergenerational chains.

Intellectual Groups
Master-Pupil Chains -----→ Field of intellectual activity = Inner Experience
Rivalries

Mind or mentality, a train of thinking in a particular body, is constituted by one's personal history in a chain of social encounters.

The meshing of chains of local encounters [interaction rituals (IRs)] = Interaction Ritual Chains (IRCs).

Symbols are charged with meaning in IRs & IRCs.

The shape of a network and where individuals happen to be within it determine what they can do, what they can think, and how creative they can be.

MATHEMATICS REPRESENTS NETWORKS OF IRCs

The next stage is to raise questions about whether general solutions are possible in specific areas (e.g., the quintic equation). Meta-issues arise and lead to the theory of groups. During the 1800s, self-reflexivity engages and mathematics leaves physical representation behind to an extent unknown in earlier periods (e.g.,non-Euclidean geometries), and mathematicians begin to vary the axiomatic sets of operations underlying conventional algebra (leading to the invention of alternative algebras). The abstraction-reflexivity sequence intensifies from the eighteenth century on as a consequence of the lengthening of the chain of generational continuity (reflected in the professionalization and

bureaucratization of mathematics) and flows further away than ever before from a grounding in the everyday world of objects and processes. This reinforced Platonism, and mathematics became enmeshed in an orgy of purity and transcendence. This gave rise to the psuedo-problem (from a sociological perspective) of the "unreasonable effectiveness" of mathematics (Wigner, 1960) in the "real world" of the natural sciences.. In fact, the abstraction-reflexivity process guarantees that mathematics will continue to be effective in the real world only to the extent that its historical linkages to that world are sustained and reinforced. To the extent that the problems of mathematics increasingly become the problems of mathematics itself, the effectiveness of mathematics outside of mathematics will diminish, a possibility and indeed a reality already recognized among mathematicians (e.g., Booss and Niss, 1979).

NOTES

*****This is an excerpt from the ***Stobaei Hermetica,*** part of the **Hermetica,** the ancient Greek and Latin writings ascribed to Hermes Trismegistus (Shambhala, Boston: 1993), edited and translated by Walter Scott.

**For example, an historian might have noted that 1729 was the year Edmund Burke was born, and an historian with the wits of a Ramanujan might have known that Burke was the only future British statesman born that year to a Protestant solicitor father and a Roman Catholic mother in Dublin. Or that in that same year, Leopold Joseph died and Francis III was born – a former and a future Duke of Lorraine. Newton's *Principia* was translated into English, and Emperor Yung Cheng outlawed opium smoking in China. Clara Reve, the English novelist, and Catherine the Great were born in 1729. And the Treaty of Seville was signed by France, Spain, and England.

***See the new translation of this classic by Karen Fields (1995). Her introduction is one of the clearest statements on the sociology of pure, transcendental, and religious concepts I have seen. In other words, it is one of the best statements on what "social construction" means.

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