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Mathematics has been shrouded in mystery and halos for most of its history. The reason for this is that it has seemed impossible to account for the nature and successes of mathematics without granting it some sort of transcendental status. Classically, this is most dramatically expressed in the Platonic notion of mathematics. Consider, for example, the way some scholars have viewed the development of non-Euclidean geometries (NEGs). The mathematician Dirk Struik (1967: 167), for example, described that development as "remarkable" in two respects. First, he claimed, the ideas emerged independently in Gὸttingen, Budapest, and Kazan; second, they emerged on the periphery of the world mathematical community. And the distinguished historian of mathematics, Carl Boyer (1968: 585) characterized the case as one of "startling…simultaneity." These reflect classical Platonic, transcendental views of mathematics. One even finds such views in the forms of the sociology of knowledge and science developed from the 1920s on in the works of Karl Mannheim and Robert K. Merton and their followers. Mannheim, for example, wrote in 1936 that 2+2 = 4 exists outside of history; and Merton championed a sociology of science that focused on the social system of science and not on scientific knowledge which lay outside of the influences of society and culture. There are a couple of curiosities here. In the case of non-Euclidean geometry, for example, even a cursory review of the facts reveals that NEGs have a history that begins with Euclid's commentators, runs through names like Saccheri, Lambert, Klügel, and Legendre, and culminates in the works of Lobachevsky, Reimnann, and Bolyai. Moreover, far from being independent, the latter three mathematicians were all connected to Gauss who had been working on NEGs since at least the 1820s. One has to wonder why in the face of the facts of the case Struik and Boyer chose to stress the “remarkable” and the “startling.” Even more curious in the case of the sociology of knowledge is the fact that already in his The Elementary Forms of Religious Life published in 1912, Emile Durkheim had linked the social construction of religion and the gods to the social construction of logical concepts. Durkheim’s program in the rejection of transcendence languished until the emergence of the science studies movement in the late 1960s and the works of David Bloor, Donald MacKenzie, and Sal Restivo in the sociology of mathematics.

It is interesting that a focus on practice as opposed to cognition was already adumbrated in Richard Courant’s and Herbert Robbins’ classical text titled “What is Mathematics?” (1941). It is to active experience, not philosophy, they wrote, that we must turn to answer the question “what is mathematics”? They challenged the idea of mathematics as nothing more than a set of consistent conclusions and postulates produced by the “free will” of mathematicians. Forty years later, Philip J. Davis and Reuben Hersh (1981) wrote an introduction to “the mathematical experience” for a general readership that already reflected the influence of the emergent sociology of mathematics. They eschewed Platonism in favor of grounding the meaning of mathematics in “the shared understanding of human beings…” (410). Their ideas reflect a kind of weak sociology of mathematics that still privileges the mind and the individual as the creative founts of a real objective mathematics.

Almost twenty years later, Hersh, now clearly well-read in the sociology of mathematics, wrote “What is Mathematics, Really?” (1997). The allusion to Courant and Robbins is not an accident; Hersh writes up front that he was not satisfied that they actually offered a satisfactory definition of mathematics. In spite of his emphasis on the social nature of mathematics, Hersh views this anti-Platonic anti-foundationalist perspective as a philosophical humanism. While he makes some significant progress by comparison to his work with Davis, by conflating and confusing philosophical and sociological discourses, he ends up once again defending a weak sociology of mathematics.

There is a clear turn to practice, experience, and shared meaning in the philosophy of mathematics, the philosophy of mathematics education, and among reflexive mathematicians. This turn reflects and supports developments in the sociology of mathematics, developments which I now turn to in order to offer a "strong programme" reply to the question "What is mathematics?"

We are no longer entranced by the idea that the power of mathematics lies in formal relations among meaningless symbols, nor are we as ready as in the past to take seriously Platonic and foundationalist perspectives on mathematics. We do, however, need to be more radical in our sociological imagination if we are going to release ourselves from the strong hold that philosophy has on our intellectual lives. Philosophy, indeed, can be viewed as a general Platonism and equally detrimental to our efforts to ground mathematics (as well as science and logic) in social life.

How, then, does the sociologist address the question, What is mathematics? Technical talk about mathematics – trying to understand mathematics in terms of mathematics or mathematical philosophy – has the effect of isolating mathematics from the turn to practice, experience, and shared meaning and "spiritualizing" the technical. It is important to understand technical talk as social talk, to recognize that mathematics and mathematical objects are not (to borrow terms from the anthropologist Clifford Geertz' analysis of speech) simply "concatenations of pure form," "parades of syntactic variations," or sets of "structural transformations." To address the question "What is mathematics?" is to reveal a sensibility, a collective formation, a worldview, a form of life. This implies that we can understand mathematics and mathematical objects in terms of a natural history, or an ethnography of a cultural system. We can only answer this question by immersing ourselves in the social worlds in which mathematicians work, in their networks of cooperating and conflicting human beings. It is these "math worlds" that produce mathematics, not individual mathematicians or mathematicians' minds or brains.

Mathematics, mathematical objects, and mathematicians themselves are manufactured out of the social ecology of everyday interactions, the locally available social, material, and symbolic interpersonally meaningful resources. All of what I have written in the last two paragraphs is captured by the short hand phrase, "the social construction of mathematics." This phrase and the concept it conveys are widely misunderstood. It is not a philosophical statement or claim but rather a statement of the fundamental theorem of sociology. Everything we do and think is a product of our social ecologies. Our thoughts and actions are not products of revelation, genetics, biology, or mind or brain. To put it the simplest terms, all of our cultural productions come out of our social interactions in the context of sets of locally available material and symbolic resources. The idea of the social seems to be transparent, but in fact it is one of the most profound discoveries about the natural world, a discovery that still eludes the majority of our intellectuals and scholars.

What is mathematics, then, at the end of the day? It is a human, and thus social, creation rooted in the materials and symbols of our everyday lives. It is earthbound and rooted in human labor. We can account for the Platonic angels and devils that accompany mathematics everywhere in two ways. First, there are certain human universals and environmental overlaps across biology, culture, space, and time that can account for certain "universalistic" features of mathematics. Everywhere, putting two apples together with two apples gives us phenomenologically four apples. But the generalization that $2+2 = 4$ is culturally glossed and means something very different in Plato, Leibniz, Peano, and Russell and Whitehead. Second, the professionalization of mathematics gives rise to the phenomenon of mathematics giving rise to mathematics, an outcome that reinforces the idea of a mathematics independent of work, space-time, and culture. Mathematics is always and everywhere culturally, historically, and locally embedded. There is, to recall Spengler, only mathematics and not Mathematik.

The concept-phrase "mathematics is a social construction" must be unpacked in order to give us what we see when we look at working mathematicians and the products of their work. We need to describe how mathematicians come to be mathematicians, the conditions under which mathematicians work, their work sites, the materials they work with, and the things they produce. This comes down to describing their culture – their material culture (tools, techniques, and products), their social culture (patterns of organization – social networks and structures, patterns of social interaction, rituals, norms, values, ideas, concepts, theories, and beliefs), and their symbolic culture (the reservoir of past and present symbolic resources that they manipulate in order to manufacture equations, theorems, proofs, and so on). This implies that in order to understand mathematics at all, we must carry out ethnographies – studies of mathematicians in action. To say, furthermore, that "mathematics is a social construction" is to say that the products of mathematics – mathematical objects – embody the social relations of mathematics. They are not free standing, culturally or historically independent, Platonic objects. To view a mathematical object is to view a social history of mathematicians at work. It is in this sense that mathematical objects are real.

Arithmetic, geometry, and the higher mathematics are produced originally by arithmetical or mathematical workers and later on by professional mathematicians. Ethnographies and historical sociologies of mathematics must, to be complete, situate mathematics cultures in their wider social, cultural, and global settings. They must also attend to issues of power, class, gender, ethnicity, and status inside and outside of more and less well-defined mathematical communities.