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Theory Of Mind, Social Science, And Mathematical Practice

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The very idea of “mathematical practice” implies, beyond the transparent social turn in philosophy, pedagogy, and didactics of mathematics, a theory of mind. Theories of mind may be the informal folk theories of our everyday lives or the more formal theories of professional students of mind. My conjecture is that folk theories of mind are at present still more influential in the work of students of mathematics and the mathematics classroom than are professional theories. The problem is that whichever theory prevails in any given setting or study, or for any given researcher, it is more likely than not to locate the mind in the brain and in the person. So one question I want to pose is: which theory or theories of mind are built into our theories and approaches to mathematical practice? If turning our attention to mathematical practice as opposed to focusing on questions of foundations is a turn to the social, perhaps we should be alert to the possibility of a social turn in our theories of mind.

Theories Of Mind

Traditionally, theories of mind, primarily coming out of philosophy and psychology, have been asocial. These theories include Hegelian mentalism, idealism, materialism, dualism, various forms of monism, and variations on these themes including Cartesian, bundle, interactionist, parallelist, behaviorist, logical behaviorist, functionalist,

phenomenological, central state or identity theories, and various attribute theories (Armstrong, 1968; Priest, 1991). One of the most intriguing aspects of how philosophers and psychologists think about minds is the effort to explore the nature of the human mind by imaginings about brains in vats, armadillo minds, thinking bats, and Martian brains. No wonder we can't find social human beings anywhere in these theories!

These traditional and prevailing approaches to mind and mentality in general center on the brain. Mentality is viewed as either caused by or identical with brain processes. Given this perspective, John Searle (1984:18) could argue that "Pains and all other mental phenomena are just features of the brain (and perhaps the rest of the central nervous system)." But Durkheim's analysis of different degrees of social solidarity and the social construction of individuality suggests a culturological conjecture on pain: the extent to which a person feels pain depends in part on the kind of culture s/he is a product of, and in particular the nature and levels of social solidarity in the social groups s/he belongs to. Furthermore, the symbolism of the pain experience in its cultural context is also a determinant of felt pain. "Pain" has a context of use, a grammar. Such a conjecture was indeed already formulated by Nietzsche (1956/1887: 199-200) in *The Genealogy of Morals*. Wittgenstein's (1953) writings on pain in his *Philosophical Investigations* provide additional ingredients for a social theory of mind based on the role of language in our pain narratives. But Searle, while he invokes the social, does not know how to mobilize it theoretically, and so argues that consciousness is caused by brain processes. We will see as we proceed why this claim that has seemed so reasonable for so long must be reconsidered in light of what we know about the relationship between social life and consciousness, and what we are learning about social life and the brain.

Cognitive psychologists tend to view the mind as a set of mental representations. These representations are then posited to be causes behind an individual's ability to "plan, remember and respond flexibly to the environment" (Byrne, 1991: 46). Cognitivists also have a tendency to equate cognition and consciousness. But Nietzsche long ago had the insight that consciousness is a social phenomenon. He was one of a number of classical social theorists who had pioneering insights into the social nature of mentality.

We can approach the history of discourse on mind in terms of (1) the conflict between rationalists (intellectual descendants of Descartes and Leibniz) and empiricists (followers of Locke, Berkeley, and Hume); (2) the behaviorist challenge to the radical empiricists by Watson and others, and the challenge in turn to the behaviorists by the ethologists (Lorenz, Tinbergen, and von Frisch); and (3) the Kantian counterpoint to empiricism, represented in our own time, for example, by Jerry Fodor's (1983) conception of the mind as an entity possessing organizing capacities and an innate "language of thought."

Why is it we "locate" mind, thinking, and consciousness "inside" heads? Certainly in the West, mentalities and the emotions have been associated with the brain and the heart since at least the time of the ancient Greeks. More recently, localizationalist physicians and neuroscientists have reinforced the idea that mentalities are "in the head" (Star, 1989). On the other hand, in sociological perspective, mentalities are not produced out of or in states of consciousness; they are not products, certainly not simple products, of the evolution of the brain and brain states. Rather, they are by-products or correlates of social interactions and social situations. This implies that

the “unconscious” and the “subconscious” are misnomers for the generative power of social life for our mentalities – and our emotions. There is no more an unconscious than there is a God, but there are cultural mechanisms for translation and transference that point us to referents that do not exist. The thesis here is that social activities are translated into primitive thought “acts,” and must meet some filter test in order to pass through into our awareness (cf. Wertsch, 1991: 26-27; and see Vygotsky, 1978, 1986; and Bakhtin, 1981, 1986). Vygotsky and Bakhtin should be considered independent inventors of the modern social theory of mind alongside their contemporary, G.H. Mead. Wertsch (1991: 14) stresses that mind is mediated action, and that the resources or devices of mediation are semiotic. Mind, he argues, is socially distributed mediated action.

Getting To The Beginning Of Our Story

In 1939, C. Wright Mills (1963) argued that the sociological materials relevant to an understanding of mind had not been exploited by sociologists. Mills had in mind in particular the work of the American philosopher and social theorist, George Herbert Mead. Fifty years later, Randall Collins (1988) could still write that Mead’s writings on the sociology of mind were underdeveloped and unexploited. And as I write these words more than a decade later, the same claim can be made. Indeed, the failure of sociologists to pick up the track of Mead’s social theory of mind was underscored by the publication of a social theory of mind guided by Mead’s work written not by a sociologist but by a neuroscientist/psychiatrist (Brothers, 1997).

Resources For A Sociological Theory Of Mind

The basic resources I draw on for constructing a sociological theory of mind include but are not limited to the following: the concepts of *collective representations* (Durkheim) and *generalized other* (Mead); Goffman's (1974) *theory of frames* (cf. Wertsch, 1991 on *recontextualization*); the literature on *culture and thought* (Levy-Bruhl, 1985/1926; Levi-Strauss, 1966; Goody, 1977; Cole and Means, 1981); studies of the evolution of *human language and its social context* (Caporael et al., 1989), and studies *on the relationship between social relations and rule-governed systems such as language* (Caporael, 1990: 10-11). Researchers in artificial intelligence have been increasingly incorporating into their work the idea that *AI machines have to be programmed with "cultures"* (e.g., Normal and Rumelhart, 1975; Keesing, 1987: 381). By the 1990s, these and related ideas had coalesced into efforts to build affective computers and social robots (e.g., Picard, 1997; Breazeal, 2002; Restivo, 2001).

It is also important to register in these early moments of this effort in theory construction that sociology has something to say about the brain. Clifford Geertz (1973: 76) has pointed out that the brain is "thoroughly dependent upon cultural resources for its very operation; and those resources are, consequently, not adjuncts to but constituents of, mental activity..." Indeed, DeVore (Geertz, 1973: 68) has argued that primates literally have "social brains." The evidence for this conjecture in humans has been accumulating in recent years along with a breakdown of the brain/mind/body divisions (e.g., Brothers, 1997; Pert, 1997).

Our understanding of mentalities has been obstructed by some deeply ingrained assumptions about human beings. One is that affect and cognition are separate and separated phenomena. This division is breaking down (e.g., Zajonc, 1980, 1984; Gordon,

1985; Damasio, 1994; and Pert, 1997), and will have to be eliminated as part of the process of constructing a sociology of mind. Another assumption is that learning and cognition can be decontextualized. I argue with other social scientists, by contrast, that learning and cognition are linked to specific settings and contexts, that is, they are indexical. Their long-term efficacies are in fact dependent on contextual recurrence, contextual continuity, and recursive contextualizing. The latter process helps explain the process of generalization without recourse to epistemological mysteries or philosophical conundrums. We live our lives by moving from home or school to home or school, from our home to our neighbor's home, from the schools we attended to the schools our children attend. Contexts repeat, imitate, suggest, overlap, impose and re-impose themselves, shadow and mirror each other, and are linked through simple and complex feedback loops. This is the structural and informational basis for the continuities in our sense of self, our memories, our thoughts. Many of the mysteries of the paranormal and our everyday experiences of déjà vu can be explained by attending to these features of context.

According to Astington (1996: 184), Gopnik (1996) claims there are only “three games in town” when it comes to theory of mind: theory-theory, simulation theory, and modularity theory. But Gopnik (1996: 169, 182) distinguishes “theory-formation theory” from “theory-theory.” Nonetheless, the psychologism in these theories fits the individualist bias we find in work ranging from research on children's theories of mind to social robotics. There is another game in town, however, and it goes with the sociological resources I have sketched. The alternative to children deriving their theories of mind from their direct experiences of such states, of developing such theories the ways

scientists supposedly derive their theories, or of giving rise to them innately as they mature is an enculturation theory. The prevailing theories of theory of mind emphasize development within the individual. From a sociological or anthropological perspective, theory of mind and mind itself are cultural inventions (Astington, 1996: 188). Social construction of mind has not been ignored, but it has not been as centrally represented in either mind studies or social robotics. The reason is a problem in the sociology of knowledge. It may be, for instance, that it is easier to link psychology and engineering because psychology appeals to the illusion or fallacy of introspective transparency. The problem with sociology is that while it holds unparalleled promise for social and sociable robots engineering, it is by comparison with the psychological sciences introspectively counter-intuitive and technologically sterile. These are not failures of sociology but rather failures of the sociological imagination in robot science and engineering. Similar problems accrue to educational theories in mathematics to the extent that they are grounded in traditional psychology and philosophy.

The Social Mind

The sociology of mind and thinking has a long and distinguished pedigree, yet it has until recently been virtually invisible in contemporary theories of mind (Valsiner and van der Veer, 2000). A renewed interest in mind, brain, consciousness and thinking (along with the new life evident in the search for God – the two quests are indeed related in sociology's program for the rejection of transcendence) is evident in the steady stream of books, articles, lectures, news stories, and television programs crossing today's intellectual landscapes. One of the main features of this literature is that one can see

some evidence of a sociological orientation emerging, albeit timidly and fearfully, out of the shadows.

An archaeology of these developments would reveal a “journey to the social” across the entire landscape of intellectual labor. Virtually without exception, those who undertake this journey are not sociologists or anthropologists (or more generally, social scientists) and so they stop short of their mark or otherwise abort the trip. This is, indeed, a much more treacherous journey than the Westerners’ journeys to the east which have captivated (and captured) so many Western seekers. But the very fact of the journey to the social reveals the emergence of a new discursive formation, a new episteme. This episteme is new in the sense of a birth or an originating activity, but absolutely new in the scope of its impact. Beginning in the 1840s, the West entered the Age of the Social, an era of worldview changes that will carry well into the 21st century and likely beyond before it begins to embody itself in the everyday ecologies and technologies of mind in new global configurations. In this process, what was western and European about the social will get permeated and transformed into a worldview that is less ethnocentric.

Most immediately, nothing captures the spirit of this renewal better than philosopher John Searle’s *The Rediscovery of the Mind*. Searle(1992: 128), for example, writes:

I am convinced that the category of "other people" plays a special role in the structure of our conscious experiences, a role unlike that of objects and states of affairs...But I do not yet know how to demonstrate these claims, nor how to analyze the structure of the social element in individual consciousness.

And the neuroscientist Antonio Damasio (1994: 260) writes:

To understand in a satisfactory manner the brain that fabricates human mind and human behavior, it is necessary to take into account its social and cultural content. And that makes the endeavor truly daunting.

To give one more example, consider the following remarks by Stan Franklin ((1995: 10) at the beginning of his tour of mind studies. Franklin is a mathematician and computer scientist:

Let's not leave our discussion...without pointing out its major deficiency. There's no mention of culture. How can one hope to understand mind while ignoring the cultural factors that influence it so profoundly? I certainly have no such hope. I clearly recognize that the study of culture is indispensable to an understanding of mind. I simply don't know how to gently include culture....Perhaps anthropology and sociology should share a corner with cognitive psychology.

And even in artificial intelligence research, projects from Rodney Brooks' COG (the baby robot) to the view of mentality as "physically and environmentally embedded" (Torrance, 1994), and the idea of cognition as embodied action (Varela, Thompson, and Rosch, 1991), paths are being opened for social and cultural studies of mentality.

The Sociology of Mind.

The idea that the mind is a social construction is crucial to reforming our understanding of mathematics education in the light of the sociological perspective. I come to the sociology of mind by way of the sociology of science, mathematics, and knowledge. In particular, I have been concerned over a major part of my research career with bringing mathematics down to earth. To bring mathematics out of the Platonic clouds, out of transcendental realms, is equivalent to negating the idea of "pure mind."

When, and to the extent that, mathematics becomes a functionally differentiated, institutionally autonomous social activity in any given social formation, it will begin to generate mathematics out of mathematics. The vulgar notion that "mathematics causes

mathematics" (pure mathematics) arises out of a failure to (and to be able to) recognize that in a generationally extended mathematical community (or social network of mathematicians), mathematicians use the results of earlier generations of mathematical workers and mathematicians as the (material) resources for their mathematical labors. Systematization, rationalization, generalization, and abstraction in mathematics are dependent on organizing mathematical workers in a certain way. In general, this means specialized networks and sustained generational continuity. The widely recognized significance of iteration as a factor in mathematical development and creativity is dependent on the social iteration produced by generational continuity.

For centuries, it has seemed obvious that the study of mind should be under the jurisdiction of philosophers and psychologists (in their pre-modern as well as modern guises). As the matrix of mind studies became increasingly interdisciplinary in the latter part of the last century, sociology and anthropology were notably left out in the cold. It may be that these are the only modes of inquiry that have any hope of making sense out of the chaos of claims about mind, consciousness, and even God and soul coming out of contemporary physics, astronomy, biology, artificial intelligence and the neurosciences. In 1943, Warren McCulloch and Walter Pitts helped set the agenda for an immanentist approach to mind. They claimed that "LOGIC is the proper discipline with which to understand the brain and mental activity. The brain embodies logical principles in its neurons." Durkheim had already rejected immanence along with transcendence in The Elementary Forms of (the) Religious Life. That is, he rejected in the first instance the notion that ideas such as Aristotle's categorical imperatives and Kant's categories are either (a) logically prior to experience, immanent in the human mind, or a priori; or (b)

crafted by individuals. In the second instance, he rejected the idea that there are transcendental referents (for terms, for example, such as "soul," "God," and "heaven"). The crystallization of the rejection of immanence and transcendence is one of the great on-going achievements in the history of thought. The project arguably begins as early as Socrates. Cicero said that Socrates "called philosophy down from the sky..." A more recent example of this imperative is Dirk Struik's (1986: 280) conception of the goal of the sociology of mathematics: to haul the lofty domains of mathematics "from the Olympian heights of pure mind to the common pastures where human beings toil and sweat."

Sociologists like Randall Collins and myself begin our efforts in the sociology of mind by making a simplifying assumption - that thinking is internal conversation. This poses an immediate problem. That is, given everything I have written so far, and given Wittgenstein's writings on mind and thinking, I do not want to claim that thinking (as conversation, for example) is something that happens inside heads or brains. There are efforts abroad to develop an explanation of cognition as embodied action. A theory of embodied action that is properly sociological dissolves the inner/outer dilemma and the chicken/egg problem. The chicken point of view is that there is a world "out there" with pre-given properties. These exist before and independently of the images they cast on the cognitive system. The role of cognition is to recover the external properties appropriately and accurately (Realism). From an egg perspective, we project our own world, and "reality" is a reflection of internal cognitive laws (Idealism). But a theory of embodied action explains cognition/mentality in terms that depend on having a body with a variety of sensorimotor capacities embedded in more encompassing biological, psychological,

and cultural contexts (Varela, Thompson, and Rosch, 1992). Cognition is lived; sensory and motor processes, perception, and action are not independent. This approach promises to dissolve the inner/outer dilemma, and to eliminate representational paradoxes in the theory of mind. Details on how such a perspective bears on our understanding of how we learn mathematics can be found in Stephen Lerman's (1994) work. There is already more than a hint in the embodiment imperative on how to solve the mind/body problem. We do not socialize individuals, persons, or selves as sociology textbooks and theorists are wont to claim. That claim assumes as already extant what socialization aims to create. What then is socialized (assuming that that is still an appropriate term)? Suppose that what is socialized is the brain/central nervous system, with the b/cns embedded in a social ecology, a network of social relationships and interactions? A theory constructed along some such line would eliminate the mind/body problem and make a rapprochement between neuroscience and social science a reasonable expectation.

In order to cement the relationship between embodiment and the sociology of mathematics, consider what we mean by endless counting. I am inspired here by Nietzsche, Wittgenstein, and Rotman among others. We must ask in the first place who is it that will be making the endless marks or speaking the endless numbers? It will have to be us or some surrogate. We could only relate to such a surrogate if it was like us in significant ways. If what is significant about us is our embodiment, any counting surrogate would have to be embodied. The alternative is a disembodied agent counting outside of time, space, and materiality. An embodied counter undermines Platonism and absolutism and gives us the grounds for a sociology of mathematics. Numbers then appear as things embodied humans have to make and re-make.

For the moment, I want to focus on the "phenomenology" of a certain kind of thinking experience. A sociological theory of mind must account, one way or another and sooner or later, for the experience of "inner thought." And it must do so without the assumption or claim that this experience is universal across humans and cultures. Conversation is the prototype for a certain kind and level of thinking, the kind of thinking we, initially at least, have in mind (so to speak) when we set out to construct an artificial intelligence, develop a theory of mind, or think about our own thinking. We must learn to speak out loud before we can think "silently," "in our heads." "External" speech already contains all the crucial elements of thought: significant symbols, capacity to take the stance of one's interlocutor or listener, and ability to take the role of the other and orient to the generalized other (see the discussion below).

Internal conversations do not necessarily have the same structure as external conversations. Short cuts, shunts, and short-circuits in our thinking are possible when we (as adults) are thinking smoothly. We may know almost immediately where a thought is going and whether to pursue it or switch over to another thought-track. Because we can monitor multiple thought-tracks (the dispatcher function), we can rapidly switch between alternatives, elaborations, objections, and conclusions. Thought-tracks and trains of thought connect syntactically and pragmatically in Hesse-type networks (Hesse, 1974). And words invoke other words, ideas invoke ideas, concepts invoke concepts (because of similar meanings, sounds, and/or associations). Generally, these switches, invokings, and associations occur smoothly and without the exercise of "will;" and they can produce what I call thought cascades. If the process is disrupted in any way, however, our

attention will shift, the process will slow, and we will proceed with awareness. This contributes to the illusion that we think "willfully."

If we treat thinking as internal conversation, then thinking must be constructed out of past, anticipated, and hypothetical conversations. In other words, what we think is connected to our social networks (including reference groups). Then the greater the attraction to given parts of the network, the more we will "be motivated" to think the ideas circulating in those parts.

The connections among ideas are emotional as well as associative and grammatical. Words, ideas, and images have valences. And consciousness itself is a type of emotion, attentiveness. Normally this attentiveness is very mild and attached to certain sign-relations. The level of attentiveness presumably changes as social situations (real and imagined) change. Only when the smooth and easy inference (or "next move") is blocked, or contradicted by something in the situation, does the emotion erupt into consciousness. So emotional weightings (valences) affect what a person thinks about at a particular time. These ideas are consistent with neuroscientific and sociological research that suggest the existence of a baseline emotional state.

The Generalized Other Revisited.

The generalized other is the core concept in George Herbert Mead's social theory of mind. Mead introduced the idea of the generalized other to describe that component of the self constructed out of the variety of messages we receive from the people we come into contact with. The generalized other is the source of our ability to take the roles of others, and also the source of our understanding of the "rules of the game" in everyday interaction. It is the locus of what Freud called the super-ego, which gives us

"conscience." And it is the locus of what I call "moralogics." When we reason, generalized others are with us all the way, approving and/or disapproving our every move. We always reason from a standpoint. There are many standpoints, and each is guarded by a generalized other. Operating logically means operating in terms of standard and standardized critical and reasoning apparatuses. Individuals cannot be logical or illogical. They can only be in agreement or disagreement with a community of discourse, an objectivity community, a thought collective. And patrolling standpoints is therefore a moral act. If, then, reasoning is always grounded in a standpoint, there can be no General Abstract Reasoner, no eternal, universal logic. If, furthermore, patriarchy has constructed Platonism, and relativity theory, and truth-seeking Diogenes and the propagandist Goebbels, the podiums of rationality and objectivity and the arenas of emotion, then there is good reason (from a certain standpoint, now!) to conjecture that mentality or mind is "man-made." Thinking is, therefore, on these principles, gendered. Logic is the morality of the thought collective, and carries the weight of how gender and power are distributed therein.

Neither "laws of logic" nor "laws of thought" (George Boole) are intuitive, innate, or a priori. Generalized others carry socially derived logical systems that restrict, govern, filter, direct, and cue logical speech acts. Inside every word, inside every vocabulary, inside every sentence, and inside every grammar we find discourse communities; logics are language games. It follows that our thoughts, insofar as they draw on the resources of languages, are socially textured. Here Goffman's (1974) frame analysis provides another ordering apparatus. And the distinction he attends to between conversational talk and informal talk has an analogy in thought. Just as informal talk holds the individual

together across parsing moments and breaks in continuity in social projects, and just as much of what we say in the presence of others is related to creating and sustaining social solidarity, so informal thought is about self-solidarity. Speaking, Goffman points out, "tends to be loosely geared to the world." Talk is looser. I conjecture that thinking is even looser, and more vulnerable to the processes Goffman calls keying and fabricating.

Now let us think again about moralogics. Mathematics communities are in part crucibles for refining the idea of God through exercises with infinity(ies). The most abstract efforts then turn out to be tied more or less explicitly to the God project. Boole's goal was to reduce "systems of problems or equations to the dominion of some central but pervading law." This is not a simple metaphor, for Boole was set on establishing the existence of God and a universal morality. So too Cantor's transfinite numbers are implicated in the search for a proof of the existence of God. I cannot pursue this further here, but see the appendix on mathematics and God in Restivo (1992).

Where Is Thinking?

The introduction to the social construction conjecture should make it easier to understand what I mean when I say that minds and thinking are social constructions. This conception carries with it the notion that thinking is a networked and dialogic process, a series of social acts rather than something that goes on inside isolated, independent heads and/or brains. This does not mean that heads and brains are dispensable, or that neuroscientists and psychologists have nothing to teach us about minds and thinking. But it is social relations that give rise to consciousness and thinking; the genesis of consciousness and thinking is in society not in the brain. Freestanding brains do not and cannot "become" conscious, and do not and cannot generate consciousness in some sort

of evolutionary or developmental "brains in a vat" process. Consciousness, thought, and language cannot be explained or understood independently of the understanding that human beings are fundamentally and profoundly social. More importantly, they are profoundly rhythmic, and it is the coupling of their rhythms in social interaction that produces consciousness, emotions, and communication.

Individualized thoughts must be tied to their social bases if we are to understand their genesis and nature. Communicable thoughts are, by definition, shareable and shared (Durkheim, 1961: 485). All concepts are collective representations and collective elaborations - conceived, developed, sustained, and changed through social work in social settings. Indeed, Randall Collins (1997) has shown through detailed comparative historical studies that the configurations and developments of social networks of intellectuals cause particular ideas to come into being and develop or die out. This line of thinking leads to the conclusion that it is social worlds or communities that think and generate ideas and concepts, not individuals. Social worlds do not, of course, literally think in some superorganic sense. But individuals don't think either. Rather, individuals are vehicles for expressing the thoughts of social worlds or "thought collectives." Or, to put it another way, minds are social structures (Gumpłowicz, 1905; Fleck, 1979: 39). Mentality is not a human invariant. And even vision is an activity and not a neurological event (Davidson and Noble, 1989; and see Heelan, 1983 on the social construction of perception).

In order to grasp the idea that thinking is radically social, and to keep it from slipping into some spiritual or mystical realm, or becoming an empty philosophical or theological concept, one must keep firmly focused on and fully comprehend the idea that

humans are social beings and that the self is a social structure. It is also crucial that we do not project our modern post-literate experience of mentality and mind-body duality on all humans in all times and places. "Mind" is not a cultural or human universal (cf., Olson, 1986, and Davidson and Noble, 1989).

Ritual And Cognition.

Cognition arises situationally out of the natural rituals of everyday interactions and conversations. These rituals form a chain, and as we move through this chain, we come across and use more or less successively blends of cultural capital and emotional energies (Collins, 1988: 357ff.). The concept of ritual developed in the work of Emile Durkheim can be generalized and conceived as a type of framing (following Goffman, 1974). This leads to the idea that the theory of ritual can be developed in terms of the different types of framings and reframings, which constitute our movement through interaction ritual chains. From this perspective, solidarity rituals take place in a social market that is variously stratified. Language is a product of a pervasive natural ritual (words, grammatical structures, speech acts, and framings are collective representations loaded with moral significance). The ingredients of language refer outside conversations, and their sense is their symbolic connection to social solidarity and their histories in interaction ritual chains. All thoughts take place in several modalities - visual, aural, emotional, sensual - simultaneously. Indeed, it is the socially constructed, gendered, cultured body-in-society that thinks, not the individual, or the head, brain, or mind.

We are now ready to enter the world of the sociology of mathematics. But I must stress that if we enter without at least some preliminary comprehension of the ideas that self

and mentality are social, the sociology of mathematics will seem like a voyage through the Looking Glass - without any of the charm of Lewis Carroll's guidance.

CONCLUSION: A SOCIAL CONSTRUCTIONIST PARADIGM FOR THE MATHEMATICS CLASSROOM

After all of this social theory, the mathematics educator may very well still feel at a loss as to how to translate social constructionism and its entailments into classroom practices. Whatever guidelines I have provided for such a translation may seem to be distant from the immediate classroom concerns of mathematics teachers. This is inevitable given the difficulties of coming to terms with what amounts to a new worldview. Let us see if we can come a little closer to the communication essentials of the mathematics classroom.

Given social constructionism, classroom teachers and students must learn to ask some old questions with an expectation that there will be new answers. What are numbers (and what are all the basic concepts and processes that constitute mathematics?). What is a classroom? What are teachers and students? What is learning? What is truth? What does it mean to reason? What is a proof? The trick here is to see all of these old friends as *institutions*. A number of mathematics educators have made significant progress in coming to terms with this perspective and making it accessible to and applicable within the working lives of mathematics teachers.(see, for example, Ernest, 1994; 1998; Burton, 1999).

Let us adopt the teacher's point of view as s/he enters the classroom armed with the tools of sociology. To begin with, the room is no longer filled with individual

students housing individual, freestanding brains. It is now to be seen, to be experienced, as a collectivity, variously cultured and more or less culturally homogeneous or heterogeneous. No assumptions can be made about shared paradigms, practices, and discourses (Sfard, 1994: 248). I want to speak in terms of a focus on practice as opposed to reasoning. But this is a false dichotomy. Given the social constructionist perspective, it follows that all aspects of mentality are forms of social practice. The everyday significance of this is that mathematics teachers are not engaged in “teaching” students how to think mathematically. Rather, they are initiating students into different corners of the mathematical community, and on different levels of mathematical practice. Therefore, their first concern should not be with the ontology or epistemology of mathematical objects and ideas, but rather with the practices that give rise to and sustain those objects and ideas in the lives of students.

Sfard (1994: 270) recommends (following, for example, Leibniz’s philosophy, not to mention Wittgenstein) an emphasis on using concepts in various contexts rather than trying to get students to grasp concepts immediately in the abstract. It would be fruitful if teachers in all fields recognized that learning in general emerges as we use words in different contexts and not out of brute force definitions and applications. It is not shared definitions that make communication possible but rather common practices in pursuit of shared goals.

There is the suggestion in Sfard (1994: 269-271) that mathematics students might learn more effectively by recapitulating the ways the mathematical community came to collectively grasp concepts and ideas. As a social constructionist, I would certainly advocate teaching mathematics in the context of their historical development. The

historical, social, and cultural contexts cannot be separated out from the substance of mathematical objects, concepts, and ideas. As long as these dimensions of mathematics are considered, “humanistic accessories” and “nonessential distractions” from the curriculum of training, teachers will not be motivated to carry out the requisite curriculum changes. Foundational resources for such changes are readily available in the works of Restivo (1983, 1992) on the calculus, mathematical traditions across cultures, and the networks of mathematics (math worlds), MacKenzie’s (1981) study of the social construction of statistics, and Bloor’s (1976) pioneering program for a sociology of mathematics.

Collaborations between social scientists and mathematics teachers would be useful. But this will be counterproductive unless the social scientists are mathematically literate and the mathematicians are sociologically literate. It is still the case that sociological literacy is much more rare than mathematical literacy. Sociological literacy depends on a worldview free of certain fallacies. I conclude with a list of those fallacies. These fallacies are the raw materials for a set of theorems about our nature as social beings.

- The Transcendental Fallacy (the theologian’s fallacy) is that there is a world or that there are worlds beyond our own – transcendental worlds, supernatural worlds, worlds of souls, spirits and ghosts, gods, devils, and angels, heavens and hells. There are no such worlds. They are symbolic of social categories and classifications in our earthly societies and cultures. There is nothing beyond our material, organic, and social world. Death is final; there is no soul, there is no life after death. It is also possible that the so-called “many worlds interpretation” in

quantum mechanics is contaminated by this fallacy as the result of mathegrammatical illusions. The world, the universe, may be more complex than we can know or imagine, but that complexity does not include transcendental or supernatural features.

- The Subscendental Fallacy (the logician's fallacy) is that there are "deep structures" or "immanent structures" that are the locus of explanations for language, thought, and human behavior in general. Such "structures" are as ephemeral and ethereal as transcendental and supernatural worlds. They lead to conceptions of logic, mathematics, and language as "free standing," "independent," "history, culture, and value free" statements. And they support misguided sociobiological and genetic explanatory strategies.
- The Private Worlds Fallacy (the philosopher's fallacy) is that individual human beings harbor intrinsically private experiences. The profoundly social nature of humans, of symbols, and of language argues against intrinsically private experiences (as Wittgenstein, Goffman, and others have amply demonstrated).
- The Internal Life Fallacy. When we engage in discourses about surrogate counters, imitation, and artificial creatures that mimic, we need to remind ourselves that we are working in an arena of analogies and metaphors. Such efforts carry a high emotional charge because they take place at the boundaries of our skins. Analogy and generalization, if they can be shown to have constructive scientific outcomes, need not obligate us to embrace identity in, for example, building robots. Robots will not have to have "gut feelings" in the identical sense

humans have gut feelings because they are organic machines. Even this “fact” needs to be scrutinized.

- What we “feel” is given to us by our language, our conversations, our forms of talking. At the end of the day, feelings may not at all be straightforward matters of bio-electro-chemical processes. Electro-mechanical creatures will turn out to be just as susceptible to an internal life as humans once they have developed language, conversation, and forms of talk. This implies a social life and awareness. Roboticists may already have made some moves in this direction with the development of signal schemas and subsumption-based hormonal control (Arkin, 1998: 434ff.). The development of cyborgs and cybrids may make this issue moot.
- The Psychologistic Fallacy is that the human being and/or the human brain is/are free standing and independent, that they can be studied on their own terms independent of social and cultural contexts and forces. This is also known as the neuroistic error. It encompasses the idea that mind and consciousness are brain phenomena. Human beings and human brains are constitutively social. This is the most radical formulation of the response to this fallacy. A more charitable formulation would give disciplinary credibility to neuroscience and cognitive approaches to brain studies. These approaches might produce relevant results in certain contexts. Then there might be

fruitful ways to pursue interdisciplinary studies linking the social sciences to the neurosciences.

- The Eternal Relevance Fallacy is that ancient and more recently departed philosophers should be important and even leading members of our inquiring conversations about social life. An act of intellectual courage is need to rid us of Plato and Hegel. Once they are eliminated, an entire pantheon of outmoded and outdated thinkers, from Aristotle to Kant, will disappear from our radar. This move might also go a long way eliminating the worshipful attitude intellectuals often adopt to the more productive and visible members of their discourse communities. The caveat here is that some ancient and some modern thinkers (departed ones, as well as some who are still with us) who can be claimed for philosophy are still extremely valuable for us. Marx, Nietzsche, and Wittgenstein come immediately to mind.
 - The Corollary Intellectual's Fallacy is that philosophers as philosophers (and psychologists as psychologists) have anything at all to tell us anymore about the social world. In the wake of the work of sociologists from Emile Durkheim (1995/1912) to Mary Douglas (1988), all the central problems of traditional and contemporary philosophy resolve into (not “reduce to”) problems in sociology and anthropology.

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